$\Upsilon(1s) \to \gamma f_2(1270)$ Decay

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Abstract

Decay $\Upsilon(1s) \to \gamma f_2(1270)$ is studied by an approach in which the tensor meson, $f_2(1270)$, is strongly coupled to gluons. Besides the strong suppression of the amplitude $\Upsilon(1s) \to \gamma gg, gg \to f_2$ by the mass of b-quark, d-wave dominance in $\Upsilon(1s) \to \gamma f_2(1270)$ is revealed from this approach, which provides a large enhancement. The combination of these two factors leads to larger $B(\Upsilon(1s) \to \gamma f_2(1270))$. The decay rate of $\Upsilon(1s) \to \gamma f_2(1270)$ and the ratios of the helicity amplitudes are obtained and they are in agreement with data.

The measurements

$$B(\Upsilon(1S) \to \gamma f_2(1270)) = (10.2 \pm 0.8 \pm 0.7) \times 10^{-5}, \tag{1}$$

$$B(\Upsilon(1S) \to \gamma f_2(1270)) = (10.5 \pm 1.6(stat)^{+1.9}_{-1.8}(syst)) \times 10^{-5} \tag{2}$$

have been reported by CLEO in the channel of $\Upsilon(1S) \to \gamma f_2(1270), f_2(1270) \to \pi^+\pi^-$[1] and $f_2 \to \pi^0\pi^0$[2] respectively. It is known that

$$B(J/\psi \to \gamma f_2(1270) = (1.43 \pm 0.11) \times 10^{-3}[3]. \tag{3}$$

$B(\Upsilon(1S) \to \gamma f_2(1270))$ is about one order of magnitude smaller than $B(J/\psi \to \gamma f_2(1270))$. CLEO Collaboration has reported the measurements of $B(\Upsilon(1S) \to \gamma\eta'(\eta))$ whose upper limits are smaller than $B(J/\psi \to \gamma\eta'(\eta))$ by almost three order of magnitudes[4]. In Refs.[5] the dependencies of $B(J/\psi, \Upsilon(1S) \to \gamma\eta'(\eta))$ on corresponding quark masses are found and explanation of very small $B(\Upsilon \to \gamma\ (\eta', \eta))$ is presented. The question is that comparing with $B(J/\psi, \Upsilon(1S) \to \gamma\eta'(\eta))$, why $B(\Upsilon \to \gamma\ f_2)$ is not too small. $B(\Upsilon \to \gamma\ f_2)$ has been studied by many authors. In Ref.[6] a QCD analysis for $B(\Upsilon(1S) \to \gamma f_2(1270))$ has been done. In Ref.[7] the ratio $\frac{B(\Upsilon \to \gamma f_2)}{B(J/\psi \to \gamma f_2)}$ has been studied by using soft-collinear theory and nonrelativistic QCD. In 1983 we have studied the radiative decay $J/\psi \to \gamma f_2(1270)$[8]. In this letter the same approach exploited in Ref.[8] is used to study $\Upsilon \to \gamma f_2$.

The study done in Ref.[8] is based on the arguments presented in Refs.[9] that the tensor

meson $f_2(1270)$ contains glueball components

$$|f_2>= cos\phi|q\bar{q}> + sin\phi|gg> . \tag{4}$$

Tensor glueball has been studied by many authors[10]. Lattice QCD predicts the existence of light 2^{++} glueball[11]. It is reasonable to assume that there is mixing between $f_2(1270)$ and a tensor glueball. In radiative decay $J/\psi \to \gamma f_2$ the $q\bar{q}$ component of $f_2(1270)$ is suppressed by $O(\alpha_s^2(m_c))$[9]

$$\frac{\Gamma(J/\psi \to \gamma + (q\bar{q}))}{\Gamma(J/\psi \to \gamma + (gg))} \sim \alpha_s^2(m_c). \tag{5}$$

Therefore, the glueball component of f_2 is dominant in the decay $J/\psi \to \gamma f_2$. It is the same that the glueball component of f_2 is dominant in the decay $\Upsilon(1S) \to \gamma f_2$ too. In QCD the radiative decays $J/\psi, \Upsilon \to \gamma\ f_2$ are described as $J/\psi, \Upsilon \to \gamma gg,\ gg \to f_2$. The coupling between f_2 and two gluons is written as[8]

$$G^{ab}_{\alpha\beta,\lambda_2}(x_1,x_2) =< f_{gg\lambda_2}|T\{A^a_\alpha(x_1)A^b_\beta(x_2)\}|0>= \delta_{ab}e^{\frac{i}{2}p_f(x_1x_2)}G(0)\sum_{m_1m_2} c^{2\lambda_2}_{1m_11m_2}e^{*m_1}_\alpha e^{*m_2}_\beta, \tag{6}$$

where G(0) is taken as a parameter. Using Eq.(6), the helicity amplitudes of $J/\psi \to \gamma f_2$ are presented in Ref.[8]. Replacing m_c by m_b in Eqs.(3,4,11) of Ref.[8], the helicity amplitudes of $\Upsilon(1S) \to \gamma f_2$ are obtained

$$T_0 = -\frac{2}{\sqrt{6}}(A_2 + p^2 A_1),$$
$$T_1 = -\frac{\sqrt{2}}{m_J}(EA_2 + m_f p^2 A_3),$$

$$T_2 = -2A_2, \tag{7}$$

$$E = \frac{1}{2m_f}(m_\Upsilon^2 + m_f^2), \quad p = \frac{1}{2m_f}(m_\Upsilon^2 - m_f^2), \tag{8}$$

where

$$\begin{aligned}
A_1 &= -a\frac{2m_f^2 - m_J(m_\Upsilon - 2m_b)}{m_b m_\Upsilon[m_b^2 + \frac{1}{4}(m_\Upsilon^2 - 2m_f^2)]}, \\
A_2 &= -a\frac{1}{m_b}\{\frac{m_f^2}{m_\Upsilon} - m_\Upsilon + 2m_b\}, \\
A_3 &= -a\frac{m_f^2 - \frac{1}{2}(m_\Upsilon - 2m_b)^2}{m_b m_\Upsilon[m_b^2 + \frac{1}{4}(m_\Upsilon^2 - 2m_f^2)]}, \\
a &= \frac{16\pi}{3\sqrt{3}}\alpha_s(m_b)G(0)\psi_J(0)\frac{\sqrt{m_\Upsilon}}{m_b^2},
\end{aligned} \tag{9}$$

where $\psi_\Upsilon(0)$ is the wave functions of Υ at origin. The decay width of $\Upsilon \to \gamma f_2$ is derived as

$$\Gamma(\Upsilon \to \gamma f_2) = \frac{32\pi\alpha}{81} sin^2\phi\alpha_s^2(m_b)G^2(0)\psi_\Upsilon^2(0)\frac{1}{m_b^4}(1 - \frac{m_f^2}{m_\Upsilon^2})\{T_0^2 + T_1^2 + T_2^2\}. \tag{10}$$

The ratios of the helicity amplitudes are defined as

$$x = \frac{T_1}{T_0}, \quad y = \frac{T_2}{T_0}. \tag{11}$$

The expressions of these quantities for $J/\psi \to \gamma f_2$ can be found from Ref.[8].

The wave functions of Υ or J/ψ at the origin are related to their rates of decaying to ee^+

$$\frac{\psi_\Upsilon^2(0)}{\psi_J^2(0)} = 4\frac{\Gamma_{\Upsilon\to ee^+}}{\Gamma_{J/\psi\to ee^+}}\frac{m_\Upsilon^2}{m_{J/\psi}^2}. \tag{12}$$

The parameters $sin^2\phi G^2(0)$ are canceled in the ratio

$$R = \frac{B(\Upsilon \to \gamma f_2)}{B(J/\psi \to \gamma f_2)}$$

Taking $\alpha_s(m_c) = 0.3$, $\alpha_s(m_b) = 0.18$[6], and $m_c = 1.29GeV$(experimental value is $m_c = 1.27^{+0.07}_{-0.11}GeV$[4]), $m_b = (5.04 \pm 0.075 \pm 0.04)GeV$[4] is obtained

$$R = 0.071(1 \pm 0.17) \tag{13}$$

which agrees with experimental data[4].

The ratios of the helicity amplitudes are obtained

$$x^2 = 0.058, \;\; y^2 = 5.9 \times 10^{-3}. \tag{14}$$

They are consistent with experimental values[1]

$$x^2 = 0.00^{+0.02+0.01}_{-0.00-0.00}, \;\; y^2 = 0.09^{+0.08+0.04}_{-0.07-0.03}. \tag{15}$$

Eqs.(7,9,10) show that the approach[8] used to study the decay $\Upsilon \to \gamma f_2$ leads to strong suppression by the mass of b-quark. On the other hand, eq.(14) shows that this approach leads to

$$A_2 \sim 0 \tag{16}$$

and very small $T_{1,2}$. Therefore, the amplitude T_0 makes dominant contribution to the decay rate of $\Upsilon \to \gamma f_2$. Because of Eq.(16)

$$\Gamma(\Upsilon \to \gamma f_2) \propto p^2, \tag{17}$$

Eq.(17) leads to a strong enhancement for the decay rate. The T_0 dominance has been found in Ref.[6] and $R \sim 0.059$ is obtained. In Ref.[6] $m_c = 1.5GeV$ is taken. The value used in

this study is consistent with the experimental data[4] and the amplitudes are sensitive to the value of m_c. Therefore, there is competition between the suppression and the enhancement in the decay $\Upsilon \to \gamma f_2$. In QCD $J/\psi,\ \Upsilon \to$ *light hadrons* are described as $J/\psi, \Upsilon \to 3g$ whose decay width is proportional to $\alpha_s^3 m_V$, where m_V is the mass of $J/\psi, \Upsilon$ respectively. Putting these factors together, the ratio is expressed as

$$\begin{aligned} R = \frac{B(\Upsilon \to \gamma f_2)}{B(J/\psi \to \gamma f_2)} &= \frac{\Gamma(\Upsilon \to \gamma f_2)}{\Gamma(J/\psi \to \gamma f_2)} \frac{\Gamma(J/\psi \to lh)}{\Gamma(\Upsilon \to lh)} \frac{B(\Upsilon \to lh)}{B(J/\psi \to lh)} \\ &= 1.06 \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \frac{p_\Upsilon^4}{p_J^4} \frac{m_J m_c^6}{m_\Upsilon m_b^6} \frac{[m_c^2 + \frac{1}{4}(m_J^2 - 2m_f^2)]^2}{[m_b^2 + \frac{1}{4}(m_\Upsilon^2 - 2m_f^2)]^2} \frac{(1 - \frac{m_f^2}{m_\Upsilon^2})}{(1 - \frac{m_f^2}{m_\Upsilon^2})} \\ & \frac{\{2m_f^2 - m_\Upsilon(m_\Upsilon - 2m_b)\}^2}{\{2m_f^2 - m_J(m_J - 2m_c)\}^2 + 6\frac{m_f^2}{m_J^2}\{m_f^2 - \frac{1}{2}(m_J - 2m_c)^2\}^2}, \end{aligned} \tag{18}$$

where

$$p_J = \frac{m_J^2}{2m_f}(1 - \frac{m_f^2}{m_J^2}). \tag{19}$$

The competition between the suppression and the enhancement in the decay $\Upsilon \to \gamma f_2$ makes the dependence of $\frac{B(\Upsilon \to \gamma f_2)}{B(J/\psi \to \gamma f_2)}$ on quark masses much weaker than the ratio $\frac{B(\Upsilon \to \gamma \eta'(\eta)}{B(J/\psi \to \gamma \eta'(\eta))}$[5].

In summary, the approach[4] in which $f_2(1270)$ is strongly coupled to two gluons leads to very small ratios of helicity amplitudes, x and y, and not small branching ratio of $\Upsilon \to \gamma f_2$

References

[1] S.B.Athar et al., CLEO Collaboration, Phys.Rev.**D73**, 032001(2006).

[2] D.Besson et al., CLEO Collaboration, Phys.Rev.**D75**, 072001(2007).

[3] C.Amsler et al., Particle Data Group, Phys. Lett.**B667**, 192008).

[4] S.B.Athar et. al., CLEO Collaboration, Phys. Rev. **D76**, 072003(2007).

[5] Bing An Li, Phys.Rev.**D77**,097502(2008).

[6] J.P.Ma, Nucl. Phys. **B605**,625(2001).

[7] S.Fleming, C.Lee, and A.K.Leibovich, Phys.Rev.**D71**,074002(2005).

[8] Bing An Li and Q.X.Shen, Phys. Lett.**B126**,125(1983).

[9] J.L.Rosner, Phys. Rev.**D24**,1347(1981); J.F.Donoghue, Phys.Rev.**D25**,1875(1982).

[10] V.V.Anisovich et al., Phys. Atom.Nucl. **69**,520(2006), Int.Mod.Phys.**A20**,6327(2005; F.Giacosa, T.Gutsche, V.E.Lyubovitskij, A.Faessler, Phys. Rev.**D72**,114021(2005); S.R.Cotanch, R.A.Williams, Phys.Lett.**B621**,269(2005); C.Amsler et al., Crystal Barrel Coll., Phys.Lett.**B520**,175(2001);A.Sarantsev, Nucl.Phys.**A675**,193C(2000); K.F.Liu, B.A.Li, and K.Ishikawa, Phys.Rev.**D40**,3648(1989); K.Ishikawa, I.Tanaka, K.F.Liu, B.A.Li, Phys.Rev.**D37**,3216(1988).

[11] Y.Chen et al., Phys.Rev.**D73**,014516(2006); S.A.Chin, C.Long, and D.Robson, Phys.Rev.Lett.**60**,1467(1988); T.A.DeGrand, Phys.Rev.**D36**,3522(1987).